Exercise 22

Find the derivative of the function using the definition of derivative. State the domain of the function and the domain of its derivative.

$$f(x) = mx + b$$

Solution

Calculate the derivative of $f(x)$ using the definition.

$$\begin{aligned} f'(x) &= \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \to 0} \frac{[m(x+h) + b] - (mx + b)}{h} \\ &= \lim_{h \to 0} \frac{(mx + mh + b) - mx - b}{h} \\ &= \lim_{h \to 0} \frac{mh}{h} \\ &= \lim_{h \to 0} m \\ &= m \end{aligned}$$

The domain of $f(x)$ is $\{x \mid -\infty < x < \infty\}$, and the domain of $f'(x)$ is $\{x \mid -\infty < x < \infty\}$. $f(x)$ and $f'(x)$ are polynomials, so any number can be plugged into them.